

The Temporal State of the Study of the Boltzmann Constant Redetermination by a Gas Cylindrical Resonator

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This report summarizes the temporal state of the investigation of the redetermination of the Boltzmann Constant by a gas cylindrical resonator in NIM. The cylindrical resonance method has been selected for this redetermination work based on the consideration of the difficulty in the accurate measurement of the diameter of a spherical resonator, the interference level of the thermal and the viscous boundary layer and the correction procedure, etc. The solution of the theoretical model of the resonance has been obtained. A number of theoretical investigations have been conducted to provide the useful information of the degree of the imperfect effect of the boundary layers, the shell motion, the opening and the conduct pipe on the side wall of a resonator, the requirement of the tolerance of the surface polishing and the roundness of a cylindrical resonator, the parallelity and the optimal diameter of the two ends of the cylinder, etc. The facility of the laser interferometry is being constructed for the accurate measurement of the longitudinal dimension of a cylindrical resonator. The procedure technical details are well known. In principle it is not difficult to limit the uncertainty contribution of the measurement of the longitudinal dimension within 1 ppm. We have designed and made a current preamplifier which was tested to produce 20 mV output to the coupled lock-in preamplifier. The main electronic parts have been coupled together and tested. A high stability thermostat, which provides the expected stability and uniformity within ± 0.1 mK at 273.16 K, was designed and fabricated depended on the previous experience of the construction of a ± 0.5 mK thermostat. A facility to provide the pure argon gas to the resonator is under fabrication. The principle is to obtain pure argon gas directly from the liquid argon so to eliminate the possible pollution. The primary concern of this investigation is to limit and to correct the arising imperfect effects. The theoretical researches have been strengthened in order to control and to correct those imperfect effects to a satisfied low level.

The effective area of primary pressure balances operated with different gases

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In order to determine the Boltzmann constant by means of the dielectric constant gas thermometer technique (DCGT), the helium pressure at a level of one part per million (ppm) must be known. It should be measured with pressure balances. In the past, several studies have shown that the effective area of piston-cylinder assemblies is dependent on the type of gas as well as on the mode of operation. Accounting for this dependence is indispensable when a relative uncertainty in pressure measurements of about 1 ppm is required.

Usually, the effective area of gas-operated piston-cylinder assemblies used in primary pressure balances is determined from the dimensional properties of the piston and the cylinder by application of the Dadson theory in which the gas flow in the piston-cylinder annular gap is treated as being viscous and in which the effective area appears as a quantity being independent of the type of gas. In reality, the gas flow along the gap is not always viscous, especially not when the pressure balance is operated in absolute mode. In this work, the effective area is analysed as a function of the flow regime for piston-cylinders having the ideal cylindrical and real shape. The theory of the transition flow regime - which is the flow regime between the viscous and the molecular flow - shows that the effective area is dependent on the gas type. The theory is applied to three real piston-cylinder assemblies having a nominal effective area of 10 cm² and 5 cm², which were characterised accurately by dimensional measurements and are used at PTB as primary pressure standards for the realisation of the pressure scale up to 2 MPa. To verify the theoretical results, cross-float experiments were carried out between the assemblies filled with different gases (N₂, He and SF₆). The effective areas calculated by means of the transition flow theory are compared with the results obtained by the cross-float experiments. The experimental results show a good consistency with the theory, which should therefore also be applied to the piston-cylinder assemblies that are involved in the Boltzmann constant experiments.

Quasisphere Volume Measurement: Progress and Plans

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The limiting uncertainty in determining the Boltzmann Constant by means of a quasi-spherical acoustic resonator is the uncertainty on measurement of the resonator's volume. NPL's plans to determine the Boltzmann Constant involve determining the volume by three independent methods involving (a) dimensional measurements (b) pyknometry and (c) electromagnetic methods. Our plans for a dimensional volume determination involve precision CMM measurements with an uncertainty of approximately 1 micron, which are calibrated by precision diameter and depth measurements with an uncertainty of approximately 0.1 micron. We will report preliminary results for our pyknometry and electromagnetic experiments on 0.5 litre quasispheres designed by Laurent Pitre at LNE/CNAM/INM and our plans for precision manufacture of quasi-spheres in collaboration with Cranfield Univeristy.

Progress Towards an acoustic/microwave Determination of the Boltzmann Constant at LNE-INM/CNAM

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We will re-determine the Boltzmann Constant k using the relation $c^2 = 5kT/(3m)$ that, in the limit of zero pressure, connects k to the speed of sound in a noble gas c , the thermodynamic temperature T , and the mass m of a gas atom. We will obtain the speed of sound by measuring the acoustic resonance frequencies of a helium-filled, copper-walled, quasi-spherical cavity of known volume V . The volume of the cavity will be determined by measuring the microwave resonance frequencies, and/or by three-dimensional coordinate measurements. If the microwave method is satisfactory, the measurement of k will be based on the ratio between the speed of sound in helium – obtained by acoustic resonance measurements – to the speed of the light – obtained by microwave resonance measurements. Here, we report the result of our two first isotherms and our primary result in the mass molar m , volume V , zero-pressure limit of c and temperature T determination.

Temperature controls and traceability for three different experiments for the determination of the Boltzmann constant.

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At present, three are the main different measurements methods operating for the determination of the Boltzmann constant: acoustic gas thermometry (AGT), Doppler broadening thermometry (DBT) and dielectric-constant gas thermometry (DCGT). All those experiments require accurate thermal controls and well defined traceability for the temperature measurements. The Contact thermometry group of INRiM is involved in all of those methods, giving significant contribution to the thermal controls and to the accurate temperature measurements. Under the iMERA Plus JRP T1.J1.4 project, our group cooperates with the Acoustic group of INRiM, with the Fundamentals of Thermometry group of PTB and with The University of Naples II and Politecnico of Milan. At INRiM we are working on the temperature control for the acoustic resonators. Two temperature controlled bath are operating to host the spheres of different dimensions and characteristics. The temperature traceability is also guaranteed by means of a direct reference to the primary national fixed points. The cooperation with PTB is aimed at the construction of a large dimensions temperature controlled liquid bath, that will host the German DCGT experiment. The activities with the two Italian universities are aimed at the construction of a special cell and its thermal controlled chamber for the DBT experiment. Also in this case accurate temperature measurements and the associated traceability will be our job. Here we present the details of the devices, some previous results and the planned activities.

Ab initio calculations of the thermophysical properties of helium

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Numerous groups of atomic and molecular physicists and chemists have calculated the interaction potential for helium atoms, with successively greater accuracy. Other groups have used analytic representations of the potential to calculate the phase shifts for helium-helium scattering, and applied the formalism of quantum statistical mechanics and kinetic theory to calculate the virial coefficients of the equation of state, the viscosity, and the thermal conductivity. Recent calculations will be reviewed

Foreword : SI units and fundamental constants;

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There is an increasing interest of the CIPM and its Consultative committees, including the CCU, in the set up of new definitions for most of the present SI units, based on fundamental physical constants. This introducing paper will present an overview of the corresponding perspectives, especially in the fields of mass, electricity, chemical and thermal measurement.

Unified analytical model for the acoustic field in a spherical or quasi-spherical cavity: effect of modal coupling due to small perturbations

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Actually, to achieve a relative uncertainty of 10^{-6} with the acoustic method for the determination of the Boltzmann constant, a detailed modelling of the acoustic field in the resonator is still required. Several phenomena must be taken into account including heat conduction, shear and bulk viscosity of the gas, the real shape of the resonator, the acoustic input impedance of small acoustic elements flush-mounted on the wall (tubes, transducers)... Significant theoretical studies have already been done in which these perturbations (some of them modelled at the lowest order of small quantities) are accounted for separately, the coupling between them being neglected.

The scope here is thus to provide a unified model for the acoustic field in the cavity including all these perturbations (modelled at a higher order if necessary) and the resulting modal coupling, and to apply it on the practical configuration previously used at the NIST: a spherical resonator filled with argon, acoustic transducers being flush-mounted on the wall.

Update on INRiM Boltzmann project

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We present recent results obtained while developing an experiment for the determination of the Boltzmann constant at INRiM. We compare the performance of spherical resonators of different volume in terms of accuracy achieved in the measurement of acoustic and microwave resonances. We illustrate the current state of advance in developing a mathematical method for calculating the perturbation of the acoustic field due to geometrical imperfections.

Determination of the Boltzmann Constant by Means of Near-Infrared Laser Absorption Spectroscopy

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We report on a new spectroscopic method for primary gas thermometry. The method consists in retrieving the Doppler width from the absorption line shape corresponding to a given vibration-rotation transition in a CO₂ gaseous sample at thermodynamic equilibrium. There is presently a strong interest in new primary thermometric methods, likely to be employed for direct and highly accurate determinations of the Boltzmann constant (k_B), in view of a possible new definition of the unit kelvin [1].

The first spectroscopic determination of k_B has been recently performed in the mid-infrared by Daussy *et al.* on the ν_2 asQ(6,3) line of the ammonia molecule ¹⁴NH₃ at a frequency of 28 953 694 MHz, using a CO₂ laser frequency stabilized against a OsO₄ line [2]. Operating at the ice melting point under low pressure conditions (between 1 and 10 Pa), in order to stay as close as possible to the Doppler limit, the French group measured the width of the absorption line as a function of the pressure. Hence, extrapolating to zero pressure, they deduced the Doppler width, which yielded the determination of the Boltzmann constant with a relative uncertainty of 1.9×10^{-4} [2]. This approach allows for a very simple spectral analysis but requires an accurate determination of the absolute pressures.

Here we present a radically different implementation of laser absorption spectroscopy for primary gas thermometry. In particular, we demonstrate that it is possible to retrieve the gas temperature from a molecular absorption profile even when the gas pressure is sufficiently high that the line shape is far from the Doppler limit, but sufficiently small that one can neglect the averaging effect of velocity-changing collisions, so that the line shape is given by the exponential of a Voigt convolution. We perform absorption spectroscopy in the near-infrared on a CO₂ gas sample at thermodynamic equilibrium using a distributed feed-back diode laser (with a mirror-extended cavity configuration), probing the R(12) component of the $\nu_1 + 2\nu_2^0 + \nu_3$ combination band.

By doing Doppler broadening measurements as a function of the gas temperature, ranging between the triple point of water and the gallium melting point, we determined the Boltzmann constant with a precision of $\sim 9 \times 10^{-5}$ [3]. These measurements were found to be insensitive to the gas pressure. More precisely, for CO₂ pressures below a few hundred Pa, we did not observe any influence from Dicke narrowing or speed-dependent effects, at the present precision level.

[1] B. Fellmuth *et al.*, *Meas. Sci. Technol.* 17, R145 (2006).

[2] C. Daussy *et al.*, *Phys. Rev. Lett.* 98, 250801 (2007).

[3] G. Casa *et al.*, submitted to *Phys. Rev. Lett.* (Dec. 21, 2007).

2007 DCGT Temperature Scale of PTB and the Properties of Helium

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The establishment of a new temperature scale in the range from 4 K to 26 K in 2007 using dielectric-constant gas thermometry (DCGT) is described. Compared with the 1996 DCGT scale, the uncertainty with respect to thermodynamic temperature could be decreased by a factor of about two. A detailed uncertainty budget is presented and the progress in determining the different measuring quantities (capacitance changes, pressure, temperature) is explained. The uncertainty estimates are supported by several comparisons with literature data: Constant-volume temperature scale NPL-75 of the National Physical Laboratory, being the basis of the currently valid International Temperature Scale ITS-90 in this temperature range, experimental and theoretical data for the second density virial coefficient of helium-3 and helium-4, experimental results for the third density virial coefficient, and the CODATA value for the Boltzmann constant.

New data concerning the properties of helium-3 and helium-4 are presented concerning the second and third density virial coefficient as well as the polarizability. The uncertainty of the results for the polarizability is limited by the uncertainty of the thermodynamic reference given by the NPL-75 copy available at PTB, corresponding to a relative uncertainty of 30 ppm. Nevertheless, the value obtained for helium-3 is the first one on this uncertainty level. Above 3.4 K, the agreement of the results for the DCGT temperatures with the NPL-75 and the virial coefficients with literature data is surprisingly good for both isotopes. But below 3.4 K, deviations both concerning the polarizability and the virial coefficients are observed for helium-4. This could be explained by a special attractive bosonic interaction between the helium-4 atoms.

The new results obtained with DCGT at low temperatures are encouraging concerning the determination of the Boltzmann constant at the triple point of water because they verify the potential of this primary-thermometry method. The progress directed to the determination of the Boltzmann constant is described with respect to the following activities: development of new measuring capacitors (cylindrical and ring cross capacitors, the latter as originally developed at NIST), design of a high-precision thermostat (application of the experience gained at and cooperation with INRiM), development of a measuring system, which enables the comparison of two cylindrical capacitors and two ring cross capacitors. The use of quite different capacitors will aid in investigating systematic error sources as especially the influence of the effective compressibility.

Optical measurement of the Boltzmann constant at the 10^{-5} level.

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We report our progress in the direct determination of the Boltzmann constant by laser spectroscopy. The value of k_B is inferred from the Doppler profile of the linear absorption line in an ammonia vapour. Ammonia is contained in an absorption cell located inside a thermostat operating with an ice-water mixture at a temperature around 273.15 K referenced to the triple-point of water. We have recorded the Doppler profile of the asQ(6,3) rovibrational line in the ν_2 band of $^{14}\text{NH}_3$, at $\nu = 28\,953\,694$ MHz. Our earlier measurements yielded to a value of the Boltzmann constant with an uncertainty of 2×10^{-4} [1]. Recent developments of the experimental set-up and a new approach to the data processing using a Voigt line shape will be described. The new determination of k_B has a relative uncertainty at the 10^{-5} level and possible systematic errors are presently explored to go beyond.

[1] C. Daussy, M. Guinet, A. Amy-Klein, K. Djerroud, Y. Hermier, S. Briaudeau, Ch.J. Bordé, and C. Chardonnet, Phys. Rev. Lett. 98, 250801 (2007).

The Spanish Acoustic Resonator, On the Road

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In 2005 it was decided to join the experience of the University of Valladolid (Uva) in the field of acoustic measurements and the experience of the Centro Español de Metrología (CEM) in thermometry to transform the acoustic resonator of Uva, that was being used for thermophysical measurements, into a primary thermometer with the target to measure the value of the Boltzmann constant.

The progress of the works carried out and the plans for a very next future will be showed. The facilities that are being setting up and the improvements of the existing ones will also be presented together with the results that are being obtained.

Quantum theory of the Boltzmann constant determination by optical methods

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The thermal motion of atomic or molecular species results in a Doppler shift of resonances with interacting light. It has therefore been suggested [1] and demonstrated [2,3] that the Boltzmann constant could be obtained in a very direct way by an accurate measurement of the Doppler width of spectral lines. A generalized line shape will be derived for these lines from the thermal Green function for a canonical ensemble of atoms. The connection with atom interferometry will be outlined. An important theorem will be presented concerning the absence of transit broadening in a low pressure gas. The Beer-Lambert law will be extended to include the effect of saturation in optically thick media. Finally, quantum limits to such measurements will be discussed within the frame of information theory.

[1] Ch. J. Bordé, *Metrologia* 39, 435-463 (2002).

[2] C. Daussy, M. Guinet, A. Amy-Klein, K. Djerroud, Y. Hermier, S. Briaudeau, Ch. J. BORDÉ and C. CHARDONNET, *Physical Review Letters* 98, 250801(2007).

[3] C. Daussy, Ch. J. Bordé et C. Chardonnet, , *Images de la physique* 2006

Improvements in the NIST Johnson Noise Thermometry System and Implications for a Boltzmann Constant Determination

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Since 2001, NIST has been developing a Johnson noise thermometry system that uses precision waveforms, synthesized with a quantized voltage noise source (QVNS), to calibrate the electronics measuring the noise power. NIST's main goals for the Noise Thermometry program are to improve thermodynamic temperature measurements and to link thermodynamic temperature to fundamental physical constants through quantum-based electrical measurements. In particular, noise thermometry offers an electronic approach that is distinctly different from the various gas-thermometry and other approaches that have been pursued as possible methods for determining Boltzmann's constant (current relative standard uncertainty 1.7×10^{-6}).

Because we match electrical power and thermal-noise power at the triple point of water, our noise thermometer measures the ratio of the Boltzmann and Planck constants, k/h . The current relative standard uncertainty in the Boltzmann constant is 1.7×10^{-6} . Since the relative standard uncertainty in the Planck constant is much smaller than this (i.e. 5×10^{-8} , 2006 CODATA), the uncertainty in the inferred value of the Boltzmann constant is not limited by the uncertainty in the Planck constant.

Improvements to the QVNS and the cross-correlation measurement electronics have significantly reduced systematic errors and measurement uncertainty. We have improved our understanding of the contributions of preamplifier noises, transmission-line effects, common-mode signals, non-linearities of the electronic measurement system, and computational and statistical issues. In particular, a lower-noise amplifier and improved transmission-line matching has allowed us to significantly expand the bandwidth of the system and reduce the integration time required for a given uncertainty. Our improved circuit has a 600 kHz bandwidth, and has achieved a $19 \mu\text{K/K}$ Type A uncertainty with a measurement time of just 36 h. The indicated temperature is consistent with the SI-assigned temperature for the water triple point at 273.16 K, yielding a relative temperature difference of $3 \mu\text{K/K}$ (much less than the uncertainty). Our present circuit configuration could achieve a Type A uncertainty of $5 \mu\text{K/K}$ with an integration time of only 20 days, which is short compared to other noise thermometers and a 5-fold improvement over our previous JNT system.